

# Halo and RMS Beam Size Growth due to Transverse Impedance

**Slava Danilov, Jeff Holmes**  
*Accelerator Physicists*

**May 19, 2003**

## Facts:

- 1) An instability simulation of the SNS ring (using ORBIT) showed fast growth of halo above threshold.
- 2) The transverse RMS beam size grows much faster than the centroid.
- 3) Experimentally, loss begins early in the instability, when the centroid oscillation amplitude is much smaller than the vacuum chamber radius (PSR, Los Alamos).
- 4) Electron rings – with injection mismatch, large losses occur even below the instability threshold (BEP, VEPP2M, Novosibirsk).

**Question: Can we find a relation between size and centroid oscillation?**

**We present the answer in this talk.**

Our analysis uses a coasting beam model (accurate for long proton bunches).

# Analytical Model



$$F = -\operatorname{Re}\{iqID(z, \delta, \tau)Z_{\perp}(n\omega_0 + \omega_b)\delta_{\Pi}(s - s_0)\}$$

where  $F$  is the localized force,  $D$  is the dipole moment,  $Z_{\perp}$  is the transverse impedance, and  $\delta = \Delta E/E$ .

$$D(z, \delta, \tau) = d_s(\delta, \tau) \exp\{i(2\pi n \frac{z}{\Pi} + \omega_b t)\} = d_s(\delta, \tau) \exp\{i[2\pi(n + \nu_b) \frac{z}{\Pi} + 2\pi\nu_b \tau]\}$$

expresses the separation of fast and slow dependencies.  $d_s$  is slow centroid. This leads to

$$\frac{\partial d_s(\delta, \tau)}{\partial \tau} + i\Delta(\delta)d_s(\delta, \tau) = \chi \int_{-\infty}^{\infty} g(\delta)d_s(\delta, \tau)d\delta$$

which is a Landau-type equation.  $g(\delta)$  is an arbitrary energy distribution function,

$$\Delta(\delta) = \frac{\eta\delta}{\beta^2} 2\pi |n + \nu_b| \quad \text{and} \quad \chi = \frac{-qIZ_{\perp}(n\omega_0 + \omega_b)}{2\gamma m(\beta c)^2} \beta_{s_0}$$

# Solutions for Centroid, RMS, and Halo



We consider a Lorentz energy distribution, where  $\delta_0$  is the characteristic energy spread.

$$g(\delta) = \frac{\delta_0}{\pi(\delta^2 + \delta_0^2)}$$

Solution for initial conditions  $d_s(\delta, 0) = 1$ :

$$d_s(\delta, \tau) = \exp(-i\Delta\tau) - \frac{\chi(\exp(-(\Delta_0 - \chi)\tau) - \exp(-i\Delta\tau))}{\Delta_0 - \chi - i\Delta}$$

Below threshold asymptotic size  $\sigma$ :

$$\frac{\sigma^2}{d^2(0)} = \frac{N_{th}}{2(N_{th} - N)}$$

Above threshold asymptotic size  $\sigma$ :

$$\frac{\sigma^2}{\bar{d}^2} = \frac{N_{th}}{2(N - N_{th})}$$

Maximal Asymptotic Amplitude (halo)  $h$ :

Below Threshold: 
$$\frac{h}{d(0)} = \frac{N_{th}}{(N_{th} - N)}$$

Above Threshold: 
$$\frac{h}{\bar{d}} = \frac{N}{(N - N_{th})}$$

Example: 5% above threshold  $\rightarrow$  RMS size/centroid  $\approx 3$ , halo/centroid = 20

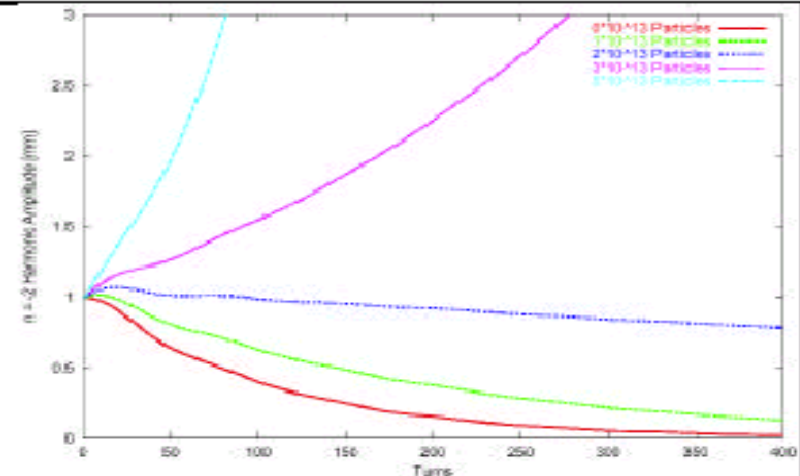
# Visualization of Analytic Formula and ORBIT (SNS Simulation Code) Benchmark



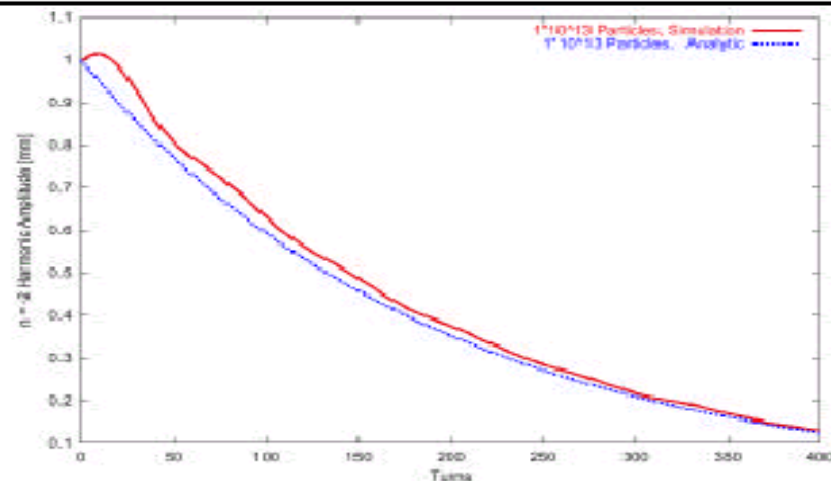
- Benchmark ORBIT with analytic calculation:
  - Straight uniform focusing lattice
  - Periodic length 40 m, tunes (1.10,1.05).
  - Localized vertical impedance ( $b/a = 2$ , second harmonic,  $Z = 2 \cdot 10^5$  Ohm/meter in detailed results shown below)
  - Coasting “pencil” beam with
    - 1 mm displacement in  $y$  (-2 harmonic);
    - Lorentz energy distribution (1 GeV, RMS width 1%, cutoff at 10%);
    - $(0-5) \cdot 10^{13}$  particles.
    - Use  $2 \cdot 10^5$  macroparticles.
- Analytic calculation with Vlasov equation and Landau damping.

# Benchmark: Evolution of Centroid Harmonics

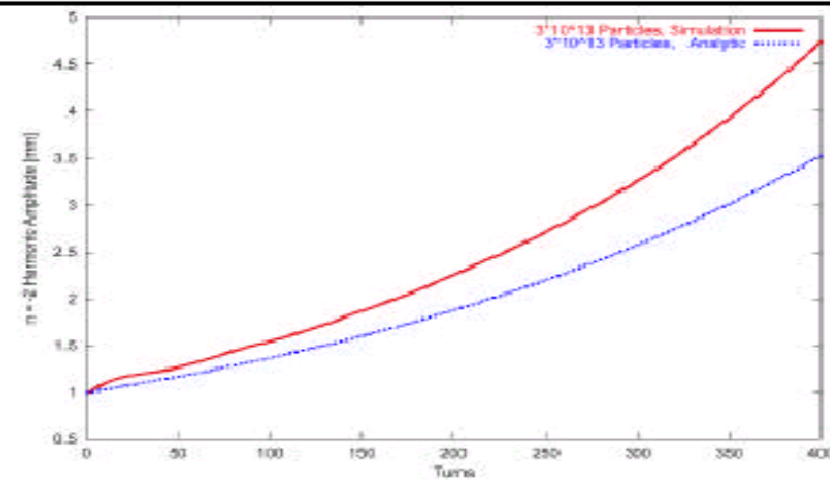
Figure 1. Evolution of  $n = -2$  harmonic for the test case: a) Simulated values for several intensities ranging  $N = (0 - 5) \cdot 10^{13}$  particles; b) Simulated (red) and analytic (blue) values for stable case with  $N = 1 \cdot 10^{13}$  particles; and c) Simulated (red) and analytic (blue) values for unstable case with  $N = 3 \cdot 10^{13}$  particles.



(a)



(b)

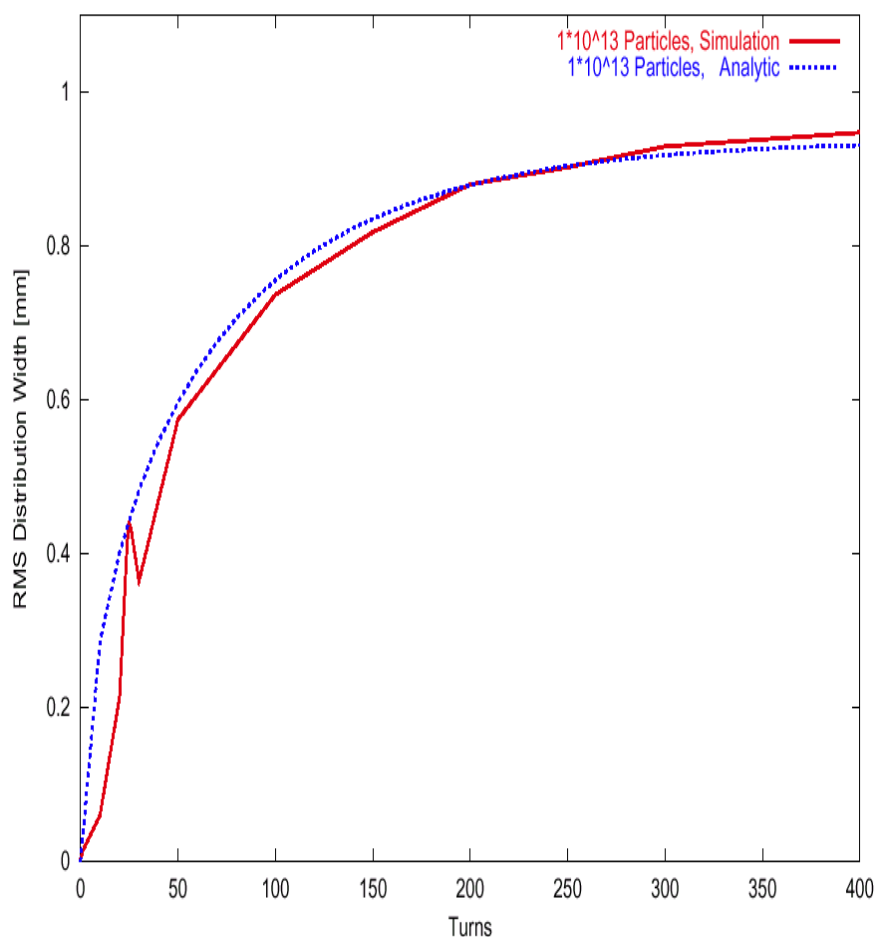


(c)

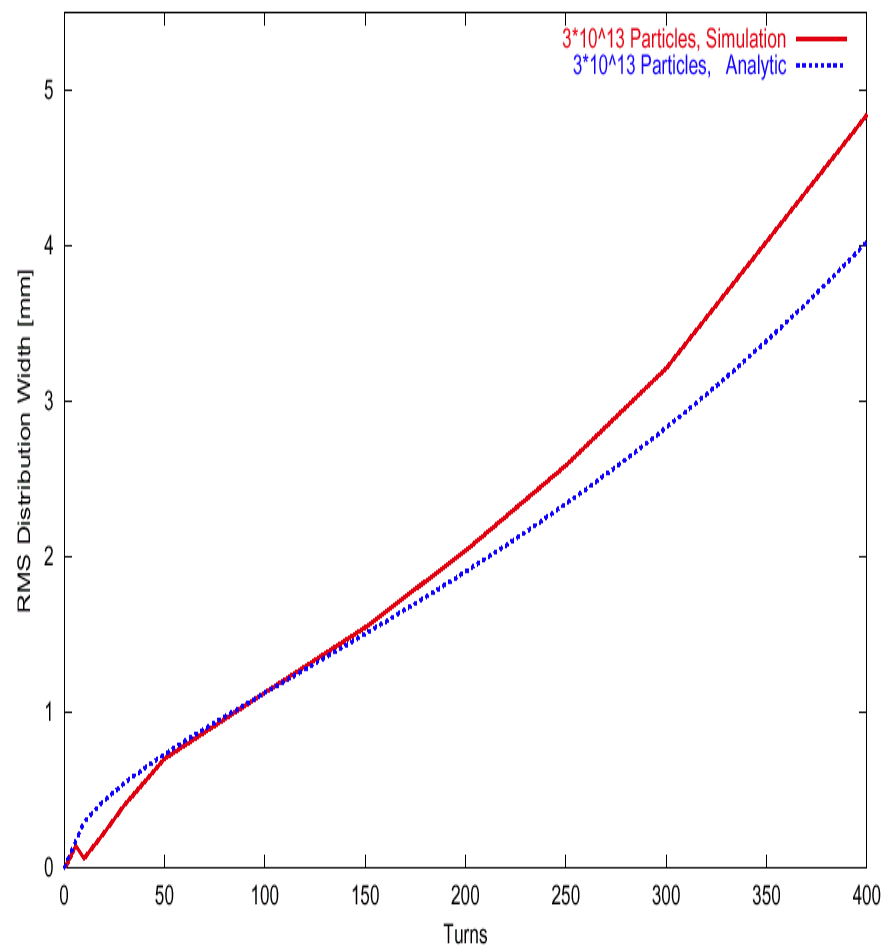
# Benchmark: Evolution of RMS Beam Widths



Stable Case:  $n = 1 \cdot 10^{13}$  protons

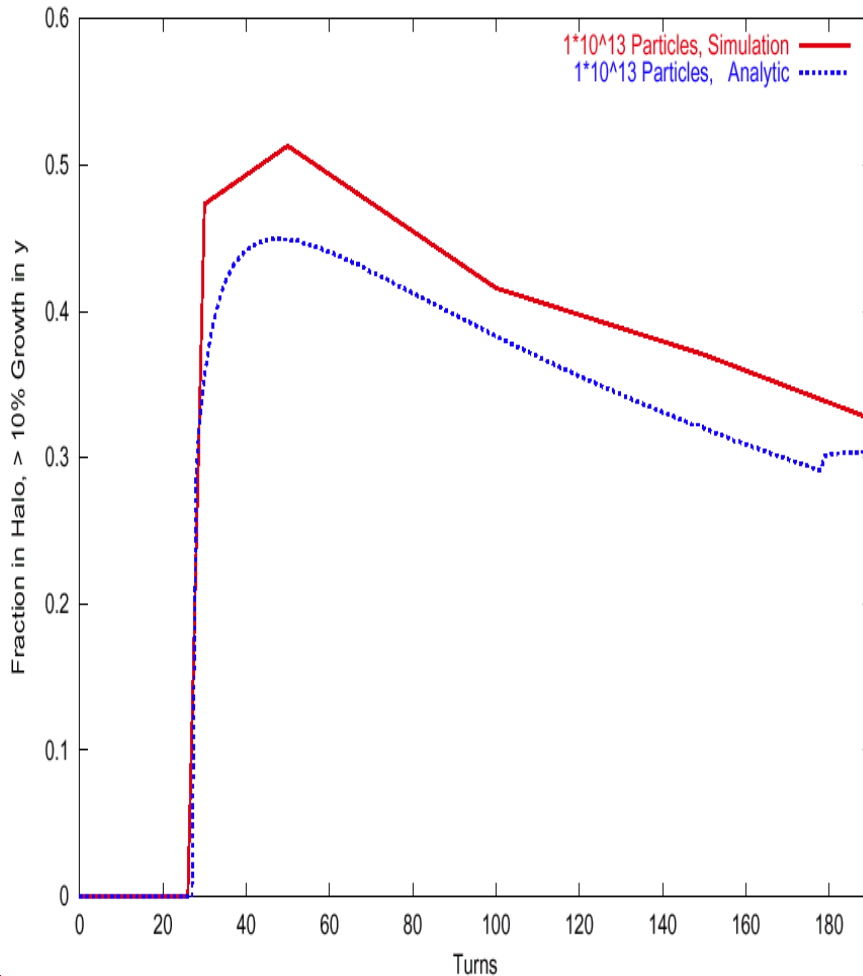


Unstable Case:  $n = 1 \cdot 10^{14}$  protons

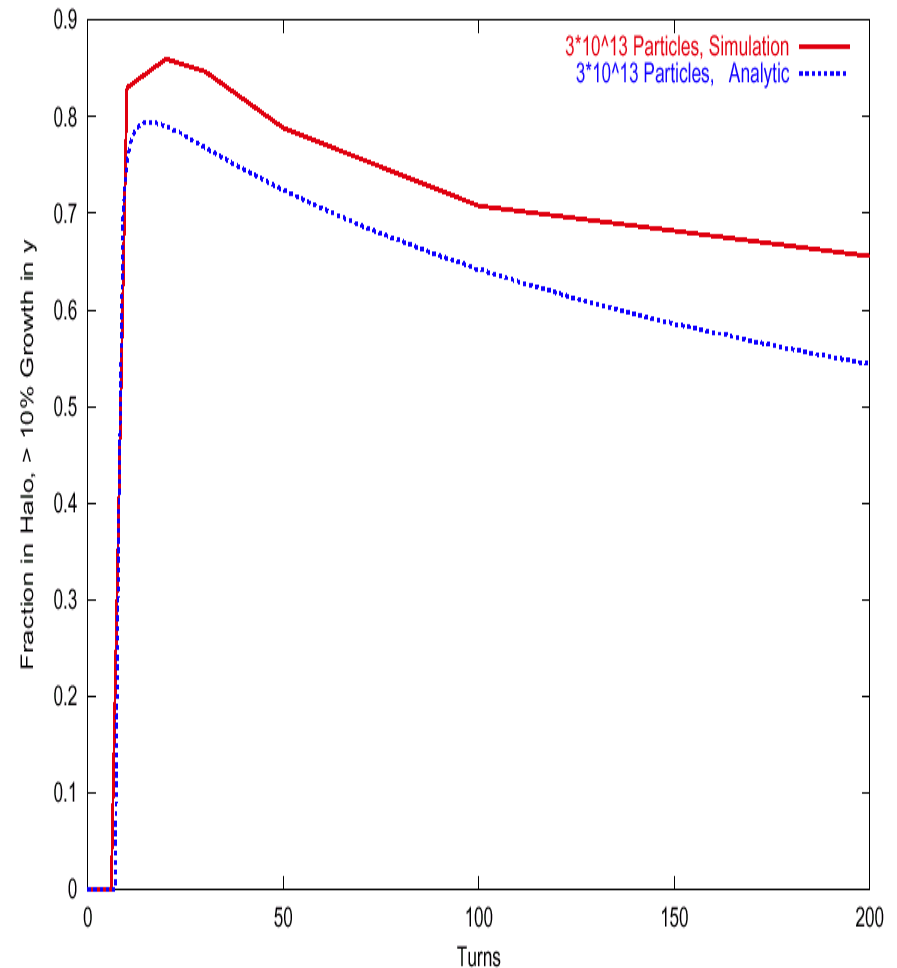


# Benchmark: Evolution of Beam Halo

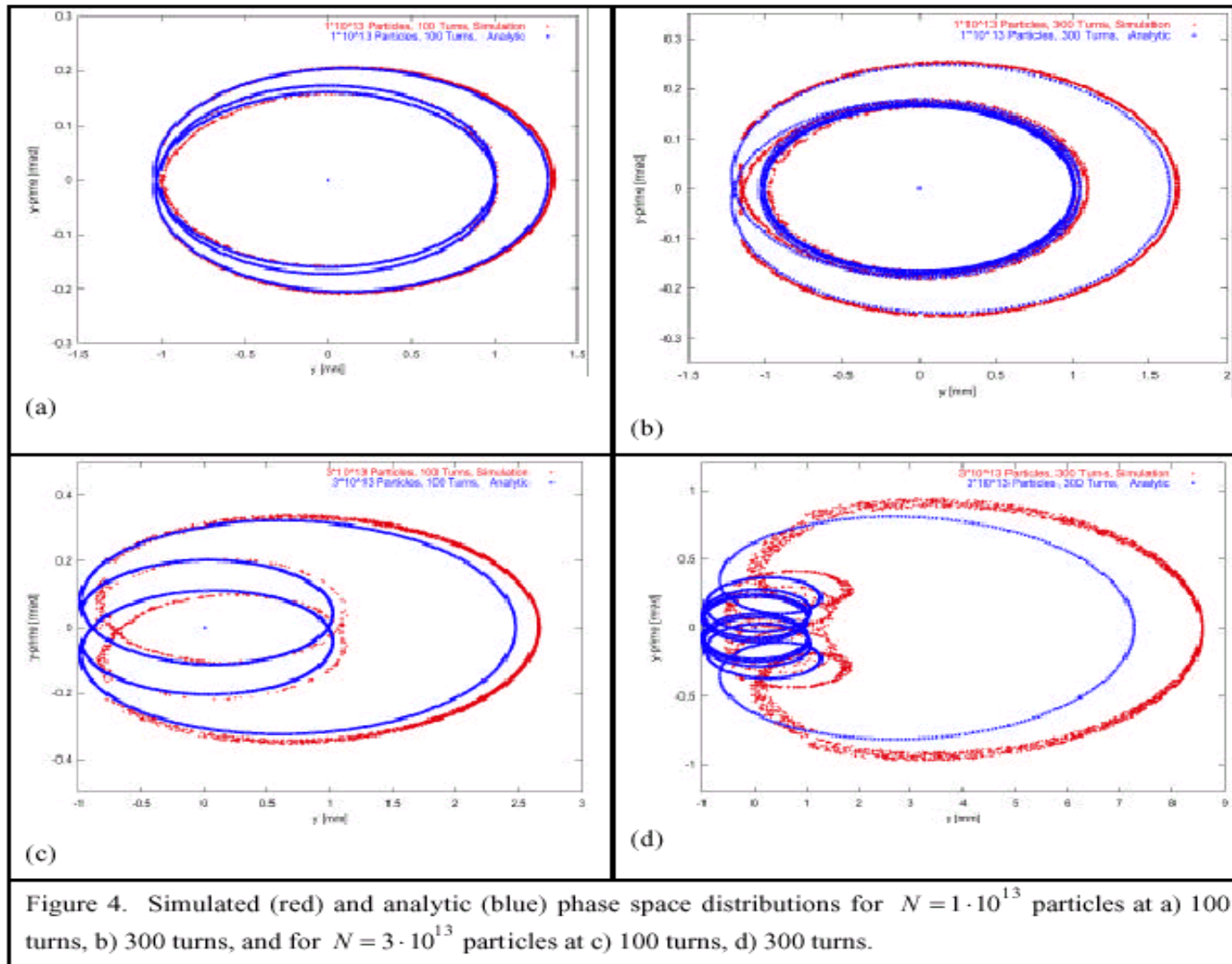
Stable Case:  $n = 1 \cdot 10^{13}$  protons



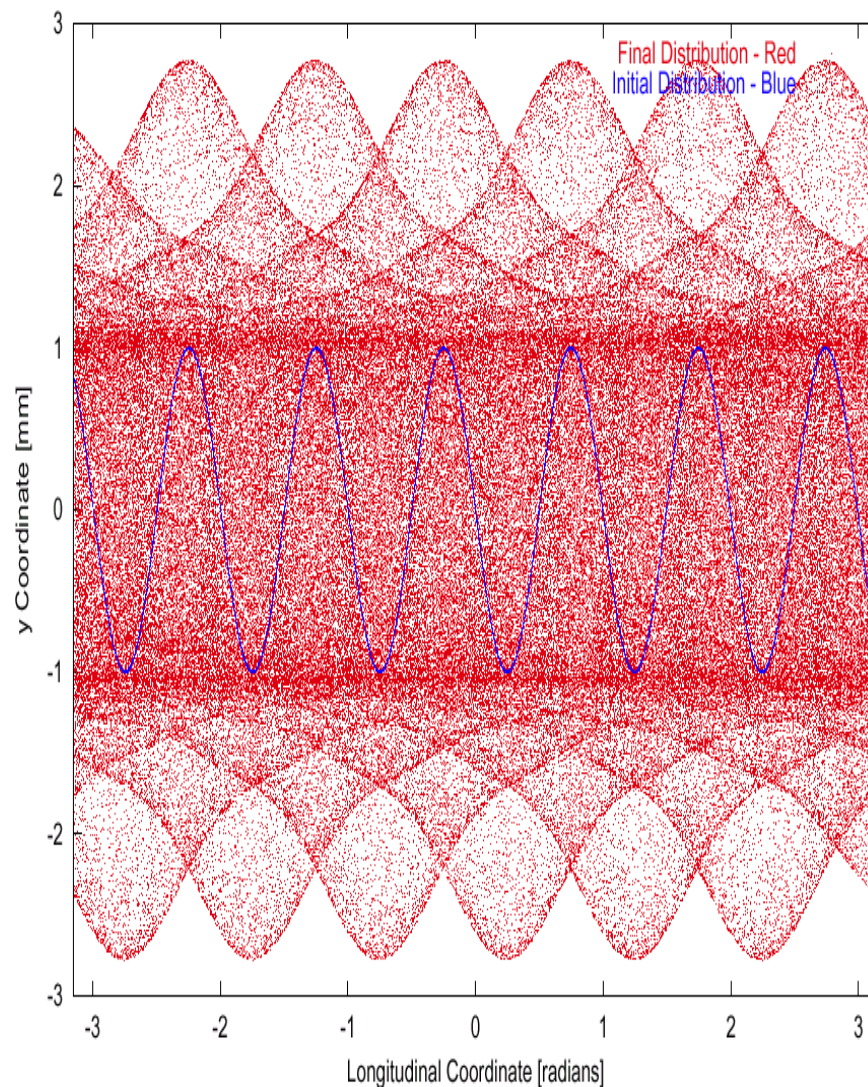
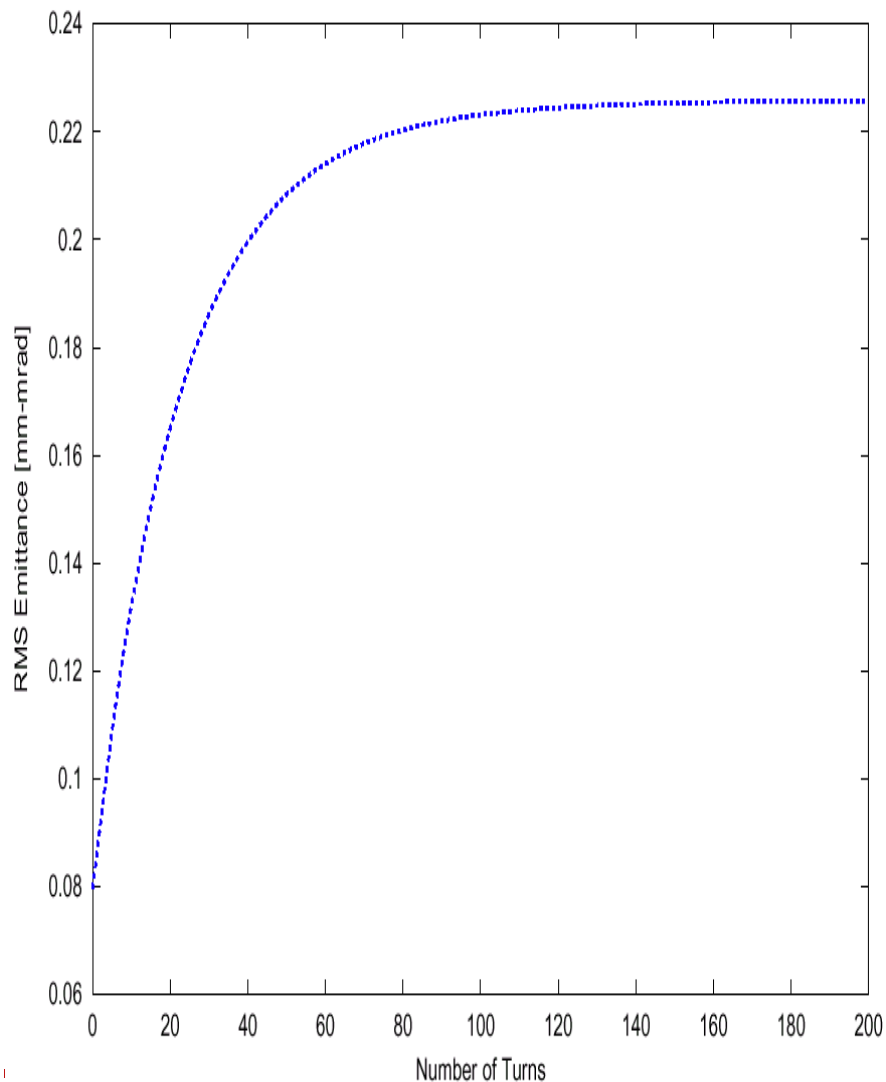
Unstable Case:  $n = 1 \cdot 10^{14}$  protons



# Benchmark: Evolution of Beam Distributions



# Benchmark: In Spite of Stability, the Beam Grows and Halo Forms (SNS Case)



# Applicability of “Pencil Beam” Model to Real Cases



- When the beam size is comparable with the centroid, results from the “pencil beam” model should be modified.
- The RMS size of the pencil beam  $\sigma_p$  should be replaced by  $\sqrt{\sigma_i^2 + \sigma_p^2}$ , where  $\sigma_i$  is the initial RMS size.
- The halo from the “pencil beam” model should be increased by the initial RMS size.

- Halo and RMS beam size grow even in stable cases. The growth is proportional to the initial centroid mismatch. If there is a noise-induced centroid offset, it will lead to halo generation enhancement due to impedance.
- The growth is fast near the instability threshold. Typical dependencies for final growth below the threshold:

$$\frac{\sigma^2}{d^2(0)} = \frac{N_{th}}{2(N_{th} - N)} \qquad \frac{h}{d(0)} = \frac{N_{th}}{(N_{th} - N)}$$

- Successful benchmarks against both analytic and experimental results are enhancing our confidence in the models.
- The results are valid for long bunches and high frequencies.
- General statement – halo always grows for resonant particles and is linked to a Landau damping mechanism.
- Open question – are there similar relations for short bunch weak head-tail or TMC type instabilities?